Indian Statistical Institute, Bangalore
B. Math.(Hons.) III Year, Second Semester
Semestral Examination
Combinatorics and Graph Theory

Time: 3 hours May 3, 2010 Instructor: N.S.N.Sastry

Maximum Marks 100

Answer all questions. Your answers should be clear, complete and to the point.

1. Show that there is a unique strongly regular graph with parameters $k = 3, \lambda = 0$ and $\mu = 1$. [8]

2. a) Define precisely a projective subplane of a projective plane.

   b) If a projective plane of order n contains a projective subplane of order m, show that either $n = m^2$ or $n \geq m^2 + m$. [3+7]

3. Given a t- (v, k, λ) design $(X, \mathbb{B})$, show that the number of blocks meeting a m- subset S of points, $1 \leq m \leq t$, in exactly n points, $1 \leq n \leq m$, depends only on m and n and not on the choice of S. [10]

4. Define the dual of a t- design. Show that a t- design, $t \geq 2$ isomorphic to its dual exists only if $t = 2$. Give an example of a 2- design which is isomorphic to its dual. [12]

5. Let A and B be subspaces of dimension k in $\mathbb{F}_q^n$, q a prime power. Determine the number of subspaces of $\mathbb{F}_q^n$ of dimension l which contain A but not B. Here, $1 \leq 2\ k < l \leq [\frac{n}{2}]$. [10]

6. a) Show that the set of zeroes of $XY = Z^2$ in the projective 3- space over $\mathbb{F}_q$ is a $(q + 1)$- arc.

   b) If q is even, determine the nucleus of the $(q+1)-$arc defined in (a).

   [12+8]

7. Define an affine plane of order n. Show that a projective plane of order n exists if, and only if, an affine plane of order n exists. [4+6+6]

8. a) Given any function f from a finite field $\mathbb{F}_q$ to itself, show that there is a unique polynomial $p(X) \in \mathbb{F}_q[X]$ of degree at most q such that $f(a) = p(a)$ for all $a \in \mathbb{F}_q$. [12]

   b) Show that $f(X) = \sum_{iz_0}^{n} a_i\ X^{p^i} \in \mathbb{F}_q[X]$, $n \geq 1$, is a permutation polynomial if, and only if, zero is the only root of $f(X)$ in $\mathbb{F}_q$. [6]

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